Math 302: Abstract Algebra
Overview Questions

These questions are designed to help you solidify your high-level understanding of the material from our course. This list is not necessarily exhaustive, but should hopefully get you thinking about the bigger picture.

- We reviewed a number of properties of integers at the start of the semester. How have the division algorithm and properties about the greatest common divisor been important to us?

- What is an equivalence relation? Why are equivalence relations important?

- What does it mean for a function from a set to itself to be an identity function?

- What does it mean for a function to have an inverse? How does the fact that a function is one-to-one and onto imply that it has an inverse? In other words, what do you need to check to confirm this fact?

- How are $\mathbb{Z}_n$ and $U(n)$ similar? How are they different?

- What are the subgroup tests? Why did we prove them?

- What are the main results that we know about cyclic groups? Can you give a rough outline of how each is proven?

- What is the significance of the Fundamental Theorem of Cyclic Groups?

- What are the main results that we know about permutations groups? Can you give a rough outline of how each is proven?

- How are the main results about cyclic groups similar to the main results about permutation groups? How are they different?

- What is an isomorphism? What is the importance of each component of the definition of an isomorphism? If two groups are isomorphic, what does this say about the groups?

- What is an automorphism? In what ways is an automorphism similar to an element of an element of D_3 or to a permutation? In what ways is it different?

- What does Cayley's theorem say? What is the significance of this theorem? Can you give an outline of the proof?

- Many of the groups we have studied so far can be understood as *transformation groups*, i.e. groups that act on sets. In what ways, for example, are D_3, $GL(2, \mathbb{R})$, $\text{Aut}(G)$, and S_n similar? How is this related to Cayley's theorem?

- What does Lagrange's theorem say? Why does it hold?

- What does it mean for a subgroup to be normal? Why is this condition important?

- What is an external direct product? What is an internal direct product? What results do we have about direct products? Can you sketch a proof of each result?

- In what ways are direct products and factor groups similar in nature to products and quotients of numbers?

- Why would we want to compute and/or study factor groups?

- What is a homomorphism? How are the properties of homomorphisms similar to those of isomorphisms? How are they different? How are the questions that we ask about homomorphisms different from the questions that we ask about isomorphisms?

- Why does a factor group satisfy the definition of being a group? What things do you need to check? Why are they true?

- Give an overview explanation of why the first isomorphism theorem for groups holds. What steps are needed to prove this theorem?

- How is the first isomorphism theorem for groups applied?

- Given a group G, in what way is the question of considering factor groups of G the same as the question of considering homomorphisms from G to other groups?

- What is the definition of a ring? How is each part of the definition important?

- Are you familiar with various examples of rings that we have studied?

- What special categories of rings have we studied? What are the definition conditions for each? How are they related to each other?

- Why does cancellation hold in an integral domain?

- What are some special properties of fields?

- What is an ideal? Why is this condition important? In what ways are ideals in rings analogous to normal subgroups in groups?

- Why does a factor ring satisfy the definition of being a ring? What things do you need to check? Why are they true?

- What is a ring homomorphism? What is the kernel of a ring homomorphism? What steps are needed to prove the first isomorphism theorem for rings?