Defn The greatest common divisor (gcd) of nonzero $a, b \in \mathbb{Z}$ is the largest integer d s.t. $d \mid a$ and $d \mid b$.

If $\gcd(a,b) = 1$, we say a, b are relatively prime.

Ex. $\gcd(30, 12) = 6$

$\gcd(8, 15) = 1 \rightsquigarrow$ 8 and 15 are relatively prime.

Defn The least common multiple (lcm) of nonzero $a, b \in \mathbb{Z}$ is the smallest positive integer m s.t. $a \mid m$ and $b \mid m$.

Ex $\operatorname{lcm}(6, 15) = 30$.

Thm For nonzero $a, b \in \mathbb{Z}$ there exist integers s and t such that $\gcd(a,b) = as + bt$.

Furthermore, $\gcd(a,b)$ is the smallest positive integers that can be written as $as+bt$, where $s, t \in \mathbb{Z}$.

proof. omitted.

Ex. $\gcd(9, 15) = 3$

$$3 = 9(\underbrace{2}_{s}) + 15(\underbrace{-1}_{t})$$

Important note: s,t are not unique. Ex: $\cancel{9}^{3} = 9(7) + 15(-4)$

just know there exists at least one such pair.

Remarks:

1. In the equation $\gcd(a,b) = as + bt$, can use Euclidean algorithm to find s and t.

$\gcd(a,b) = 1$

2. a, b relatively prime if and only if there are integers s and t such that $1 = as + bt$.

b/c 1 is the smallest positive integer.