<u>Thm</u> If $\sim$ is an equivalence relation on X, the equivalence classes of X partition X, i.e. X is the disjoint union of the equivalence classes.

<u>Proof</u> (Need: every elt. of X in exactly one equiv. class)

Let $a \in X$. Then since $a \sim a$, $a \in [a]$.

So a lives in at least one equiv. class.

To show that a lives in exactly one equiv. class, sps. $a \in [b]$ and $a \in [c]$. GOAL: $[b] = [c]$ as sets.

To show sets $X = Y$, show $X \subset Y$ and $Y \subset X$.

"contained in"

To show $X \subset Y$, let $x \in X$, and show $x \in Y$.

First, show $[b] \subset [c]$.

So, let $x \in [b]$. (NTS: $x \in [c]$ ... need to show $c \sim x$.)

Since $x \in [b]$, $b \sim x$.

Also, since $a \in [b]$ and $a \in [c]$, $b \sim a$ and $c \sim a$.

So $c \sim \underbrace{a \sim b}_{\text{symmetry}} \sim x$.

Thus, by transitivity, $c \sim x$ so $x \in [c]$.

Therefore $[b] \subset [c]$.

A similar argument shows that $[c] \subset [b]$.

Therefore $[b] = [c]$, and therefore a lies in exactly one equivalence class, as desired.

Note: The converse holds: given a partition of X, can define an equivalence relation from it.