

Induction

↳ When you want to show a property is true for all positive integers, can often use induction.

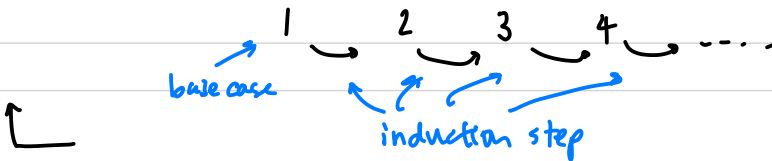
Ex Show that for all positive integers n ,

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}.$$

↳ Strategy:

1. Prove statement holds for $n=1$. *base case*
2. Assume statement holds for $n=k$. *induction hypothesis.*
3. Use assumption to show statement holds for $n=k+1$. *induction step.*

↳ These three together imply statement holds for all $n \geq 1$.



proof that $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ for all pos. integers n .

Base case: Does statement hold for $n=1$? yes: $1 = \frac{1(1+1)}{2}$ ✓

Induction hypothesis: Assume $1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$.

Induction step: (NTS ^{↙ "need to show"} $1 + 2 + 3 + \dots + k + 1 = \frac{(k+1)(k+2)}{2}$.)

$$1 + 2 + 3 + \dots + k + k + 1 = \frac{k(k+1)}{2} + k + 1 \quad (\text{by induction hypothesis})$$

$$= \frac{k^2 + k + 2k + 2}{2}$$

$$= \frac{k^2 + 3k + 2}{2}$$

$$= \frac{(k+1)(k+2)}{2}. \quad \checkmark$$

Thus, we conclude that for all positive integers n ,

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}. \quad \checkmark$$