Induction

↳ when you want to show a property is true for all positive integers, can often use induction.

Ex Show that for all positive integers n,

$$1+2+3+\dots+n = \frac{n(n+1)}{2}.$$

Strategy:

1. Prove statement holds for $n=1$. ⟵ base case
2. Assume statement holds for $n=k$. ⟵ induction hypothesis.
3. Use assumption to show statement holds for $n=k+1$. ⟵ induction step.

→ These three together imply statement holds for all $n \geq 1$.

$$1 \to 2 \to 3 \to 4 \to \cdots$$

base case

induction step

proof that $1+2+3+\cdots+n = \frac{n(n+1)}{2}$ for all pos. integers n.

Base case: Does statement hold for n=1? yes: $1 = \frac{1(1+1)}{2}$ ✓

Induction hypothesis: Assume $1+2+3+\cdots+k = \frac{k(k+1)}{2}$.

"need to show"

Induction step: (NTS $1+2+3+\cdots+k+1 = \frac{(k+1)(k+2)}{2}$.)

$$1+2+3+\cdots+k+k+1 = \frac{k(k+1)}{2} + k+1$$ (by induction hypothesis)

$$= \frac{k^2+k+2k+2}{2}$$

$$= \frac{k^2+3k+2}{2}$$

$$= \frac{(k+1)(k+2)}{2}. \checkmark$$

Thus, we conclude that for all positive integers n,

$1+2+3+\cdots+n = \frac{n(n+1)}{2}$. ✓