Brief notes on functions

Defn A function $f: X \to Y$ is a rule that assigns to each element of X a unique element of Y.

Notation: We will denote composites

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

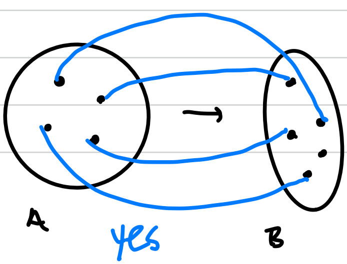
as

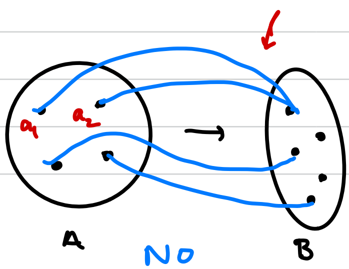
i.e. $(gf)(x) = g(f(x))$

Defn A function $f: A \to B$ is one-to-one if

$f(a_1) = f(a_2)$ implies $a_1 = a_2$.

(a.k.a. injective)





To prove f is 1-1, assume $f(x) = f(y)$ and show $x = y$.

Ex $f: \mathbb{Z} \to \mathbb{Z}$

$f(x) = 2x.$

Claim: f is 1-1.

proof: Sps $f(x) = f(y)$. Then by defn of f, $2x = 2y$.

multiply both sides by $\frac{1}{2}$, to get $x = y$. ✓

Nonex. $f: \mathbb{R} \to \mathbb{R}$

$f(x) = x^2$

For example, $f(2) = 4 = f(-2)$ but $2 \neq -2$.

Defn A function $f: A \to B$ is onto if for each $b \in B$

$\exists\, a \in A$ s.t. $f(a) = b$.

(a.k.a. surjective)

Note: "onto" vs. "into"... not the same.

To prove f is onto, select a generic element $b \in B$ and

produce a value $a \in A$ s.t. $f(a) = b$.

greatest integer $\leq x$.

Ex. $f: \mathbb{R} \to \mathbb{Z}$ $\quad f(x) = \lfloor x \rfloor$ "floor of x"

Sps $n \in \mathbb{Z}$. Then $n = \lfloor n \rfloor$ and $n \in \mathbb{R}$ so $\lfloor \cdot \rfloor$ is onto.

(Alt: $n.5$ $\quad \lfloor n.5 \rfloor = n$.)