Defn If $f: A \to B$ is both 1-1 and onto, f is called a bijection.

Ex $f: \mathbb{N} \to \mathbb{Z}$

$\mathbb{N}$	1	2	3	4	5	6	7	...
	↓	↓	↓	↓	↓	↓	↓	
$\mathbb{Z}$	0	1	-1	2	-2	3	-3	...

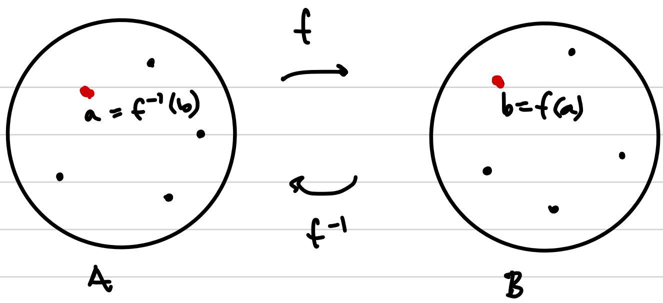
If f is a bijection, there is a one-to-one correspondence b/w the elements of A and the elements of B.

In this case, can define

$$f^{-1} : B \to A.$$

by $f^{-1}(b) = a$ where a is the element of A such that $f(a) = b$.

the value a exists b/c f is onto and it is unique b/c f is 1-1.



Note that $(f^{-1}f)(a) = a$ and $(ff^{-1})(b) = b$

for all $a \in A$ and $b \in B$.

<u>Thm</u> Sps. $f: A \to B$, $g: B \to C$, $h: C \to D$

$$A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{h} D$$

1. $h(gf) = (hg)f$

   ↳ i.e. function composition is associative (proof omitted)

2. If f and g are 1–1, so is gf. (exercise)

3. If f and g are onto, so is gf. (exercise)