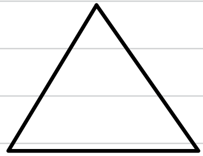
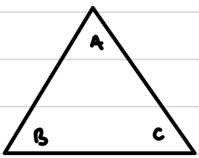
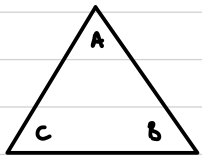
The Dihedral Group D_3

Consider symmetries of a triangle: 

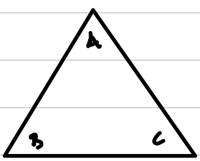


front
(starting position)

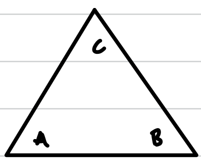


back

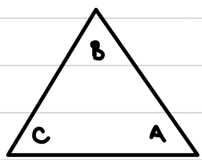
Label the symmetries:



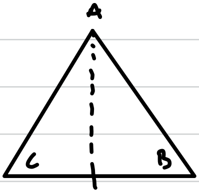
R_0



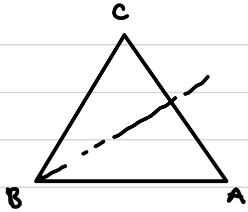
R_{120}



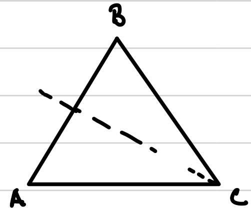
R_{240}



F_1



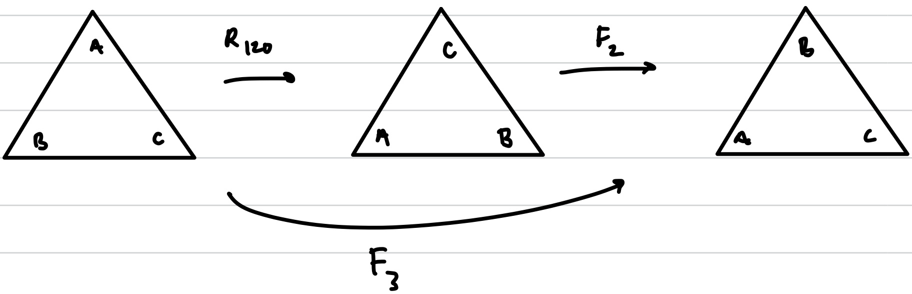
F_2



F_3

Can compose two symmetries to get a new symmetry.

Ex $\underbrace{F_2}_{2^{nd}} \underbrace{R_{120}}_{1^{st}} = F_3$

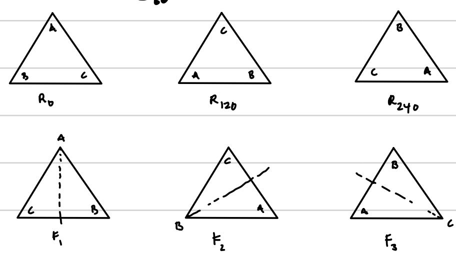


We could repeat the process with all possible pairs of symmetries...

$1^{st} \longrightarrow$

$2^{nd} \downarrow$

	R_0	R_{120}	R_{240}	F_1	F_2	F_3
R_0	R_0	R_{120}	R_{240}	F_1	F_2	F_3
R_{120}	R_{120}	R_{240}	R_0	F_3	F_1	F_2
R_{240}	R_{240}	R_0	R_{120}	F_2	F_3	F_1
F_1	F_1	F_2	F_3	R_0	R_{120}	R_{240}
F_2	F_2	$F_2 R_{120} = F_3$	F_1	R_{240}	R_0	R_{120}
F_3	F_3	F_1	F_2	R_{120}	R_{240}	R_0



Things to notice!

↳ $XR_0 = R_0 X = X$ for any symmetry X.

R_0 is the identity

↳ For each symmetry X there is some symmetry X^{-1} (the inverse of X) such that $XX^{-1} = X^{-1}X = R_0$,

e.g. $R_{120} \cdot R_{240} = R_0$ $\rightsquigarrow$ so $(R_{120})^{-1} = R_{240}$ $\quad F_2^{-1} = F_2$

↳ symmetries don't necessarily commute:

e.g. $F_1 R_{120} = F_2$

$R_{120} F_1 = F_3$

→ not the same

The group of symmetries of Δ, a.k.a. D_3,

is nonabelian.

dihedral group.

Note: we could construct D_n, symmetries of regular n-gon,

for any integer $n \geq 3$.