Group Homomorphisms

Defn Sps. G_1, G_2 are groups. A function $\varphi: G_1 \to G_2$ is a homomorphism if

$$\varphi(ab) = \varphi(a)\varphi(b) \quad \text{for all } a, b \in G_1.$$

Note: isomorphisms are homomorphisms that are 1-1 and onto. Not every homomorphism is an isomorphism.

Defn The kernel of a homomorphism φ is the set

$$\ker\varphi = \{ k \in G_1 \mid \varphi(k) = e_2 \} \subseteq G_1.$$

Ex:

↳ if φ is an isomorphism $\ker\varphi = \{e_1\}$.

$\mathbb{R}\setminus\{0\}$ under multiplication

identity $= 1$.

<u>Ex</u> $\varphi: GL(2,\mathbb{R}) \to \mathbb{R}^*$

$$\varphi(X) = \det X$$

onto,

not 1-1

operation preserving

$$\varphi(XY) = \det(XY) = \det X \det Y = \varphi(X)\varphi(Y) \checkmark$$

$$\ker\varphi = SL(2,\mathbb{R}).$$

Ex: $\varphi: \mathbb{Z} \to \mathbb{Z}$

$\varphi(x) = 0$ for all x.

exercise: check hm.

(neither 1-1, nor onto)

<u>Ex</u> $\varphi: \mathbb{Z} \to \mathbb{Z}$

$$\varphi(x) = 2x$$

Note: φ is 1-1.

$$\varphi(x+y) = 2(x+y) = 2x+2y = \varphi(x) + \varphi(y). \checkmark$$

(Not onto) $\ker\varphi = \{0\}$.