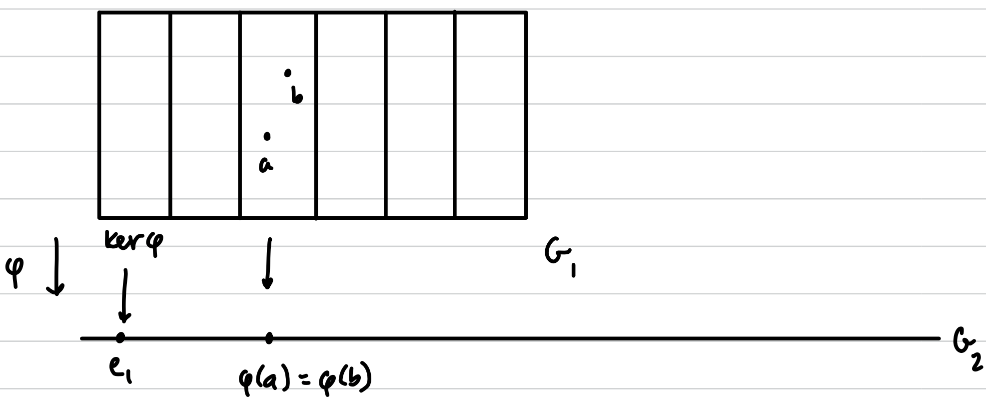
Now, can consider cosets of $\ker\varphi$ in G_1:



5. $\varphi(a) = \varphi(b) \Leftrightarrow a\ker\varphi = b\ker\varphi$

cosets

Litmus test:

$aH = bH$
$\Leftrightarrow b^{-1}a \in H$

$$\varphi(a) = \varphi(b) \text{ in } G_2 \Leftrightarrow [\varphi(b)]^{-1}\varphi(a) = e_2 \text{ in } G_2$$

$$\Leftrightarrow \varphi(b^{-1}a) = e_2$$

$$\Leftrightarrow b^{-1}a \in \ker\varphi$$

$$\Leftrightarrow a\ker\varphi = b\ker\varphi.$$

Note: φ might not be 1-1.

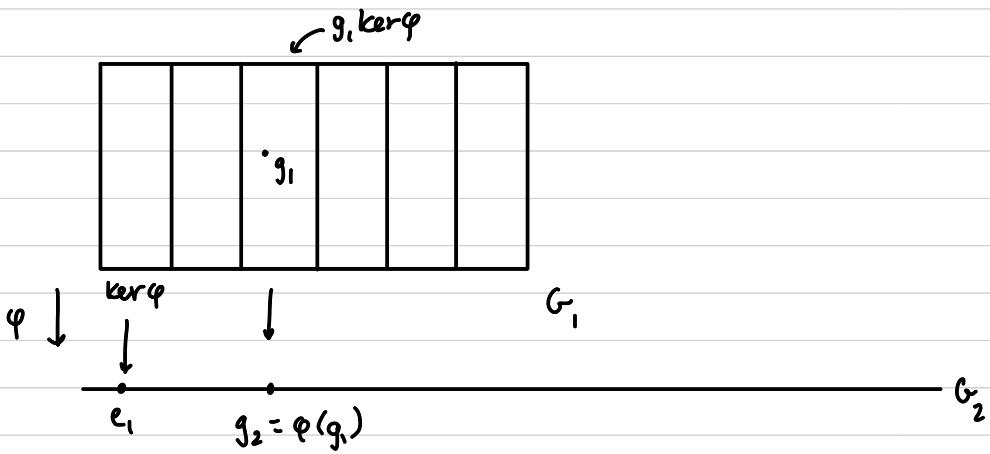
6. Sps $g_2 = \varphi(g_1)$ for some $g_1 \in G$. Define

"preimage of g_2" $\varphi^{-1}(g_2) = \{ g \in G_1 \mid \varphi(g) = g_2 \}$.

a set

Then $\varphi^{-1}(g_2) = g_1 \ker \varphi$

equal as sets



fact: φ is op. preserving

Ex. $\varphi : \mathbb{Z} \to \mathbb{Z}_4 \qquad \varphi(a) = a \bmod 4$.

$$\varphi^{-1}(3) = \{ \dots -5, -1, 3, 7, \dots \} = 3 + \langle 4 \rangle$$

g_2 $\qquad$ g_1 $\qquad$ (and $\langle 4 \rangle = \ker \varphi$)

$\varphi^{-1}(g_2) = g_1 \ker\varphi$

proof of property 6:

First, show $\varphi^{-1}(g_2) \subseteq g_1 \ker\varphi$:

Sps $h \in \varphi^{-1}(g_2)$, so $\varphi(h) = g_2$ also $= \varphi(g_1)$

This means $\varphi(h) = \varphi(g_1)$, so by property 5,

$h\ker\varphi = g_1 \ker\varphi$. Thus $h \in g_1 \ker\varphi$.

$\overset{\in}{h}$

Now, show $g_1 \ker\varphi \subseteq \varphi^{-1}(g_2)$. recall: $x \in \varphi^{-1}(g_2)$ iff $\varphi(x) = g_2$.

$k \in \ker\varphi$ .. Let $g_1 k \in g_1 \ker\varphi$. generic element

Then $\varphi(g_1 k) = \varphi(g_1)\varphi(k)$

$= \varphi(g_1) e_2$ b/c $k \in \ker\varphi$

$= \varphi(g_1)$

$= g_2$ by assumption. ✓

So: $g_1 k \in \varphi^{-1}(g_2)$. Thus $g_1 \ker\varphi \subseteq \varphi^{-1}(g_2)$.