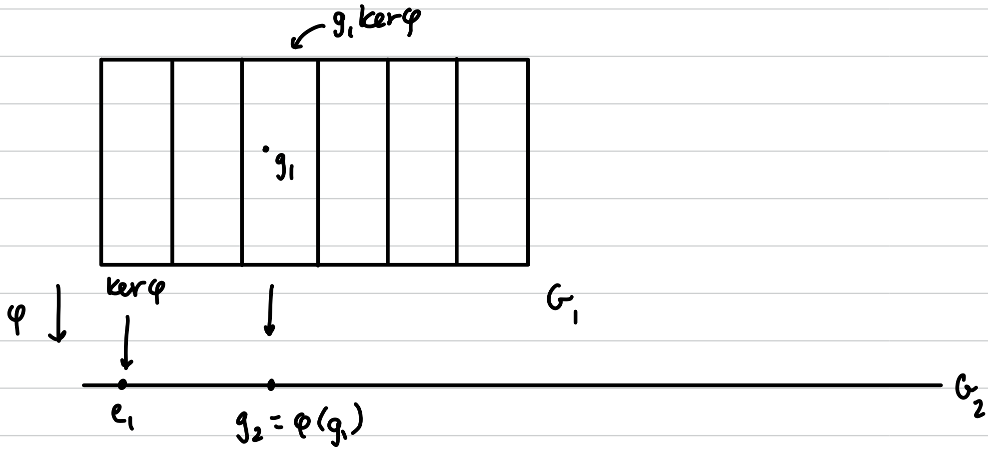
7. If $|\ker\varphi| = n$ then φ is an n-to-1 map from G_1 to G_2.



proof: by property 6, if $g_2 = \varphi(g_1)$ for some $g_1 \in G_1$, then

$$\varphi^{-1}(g_2) = g_1 \ker\varphi$$ ← a coset of $\ker\varphi$.

All cosets have same size, so if $|\ker\varphi| = n$, then

$|g_1 \ker\varphi| = n$, ie n elements map to g_2.

8. If φ is onto and $\ker\varphi = \{e_1\}$, then $\varphi : G_1 \to G_2$ is an isomorphism.

$|\ker\varphi| = 1 \Rightarrow \varphi$ is a 1-1 map. ✓