6. If $K_2 \leq G_2$, let

"preimage of K_2"

$$\varphi^{-1}(K_2) = \{ g \in G_1 \mid \varphi(g) \in K_2 \}. \subseteq G_1$$

Then $\varphi^{-1}(K_2) \leq G_1$.

(Two step subgroup test)

- $\varphi^{-1}(K_2)$ nonempty because $e_1 \in \varphi^{-1}(K_2)$.
- If $a, b \in \varphi^{-1}(K_2)$, then $\varphi(a), \varphi(b) \in K_2$

  So $\varphi(ab) = \varphi(a)\varphi(b) \in K_2$.

  Thus $ab \in \varphi^{-1}(K_2)$.
- If $a \in \varphi^{-1}(K_2)$, then $\varphi(a) \in K_2$ so $[\varphi(a)]^{-1} \in K_2$

  But $[\varphi(a)]^{-1} = \varphi(a^{-1})$ so $\varphi(a^{-1}) \in K_2$.

  Thus $\varphi^{-1}(K_2)$ is closed under inverses. ✓

normal

7. If $K_2 \trianglelefteq G_2$, then $\varphi^{-1}(K_2) \trianglelefteq G_1$.

(Normal subgroup test.)

Let $g \in \varphi^{-1}(K_2)$ and $x \in G_1$. (NTS: $xgx^{-1} \in \varphi^{-1}(K_2)$)

conjugation by element of G_2.

Then $\varphi(xgx^{-1}) = \varphi(x)\varphi(g)\varphi(x^{-1})$

in K_2 $\quad [\varphi(x)]^{-1}$

This is an element of K_2 because $K_2 \trianglelefteq G_2$.

So $xgx^{-1} \in \varphi^{-1}(K_2)$. Thus $\varphi^{-1}(K_2) \trianglelefteq G_1$.

<u>Corollary</u>: $\ker\varphi \trianglelefteq G_1$

because $\ker\varphi = \varphi^{-1}(e_2)$ and $\{e_2\} \trianglelefteq G_2$.

So $G/\ker\varphi$ is a group