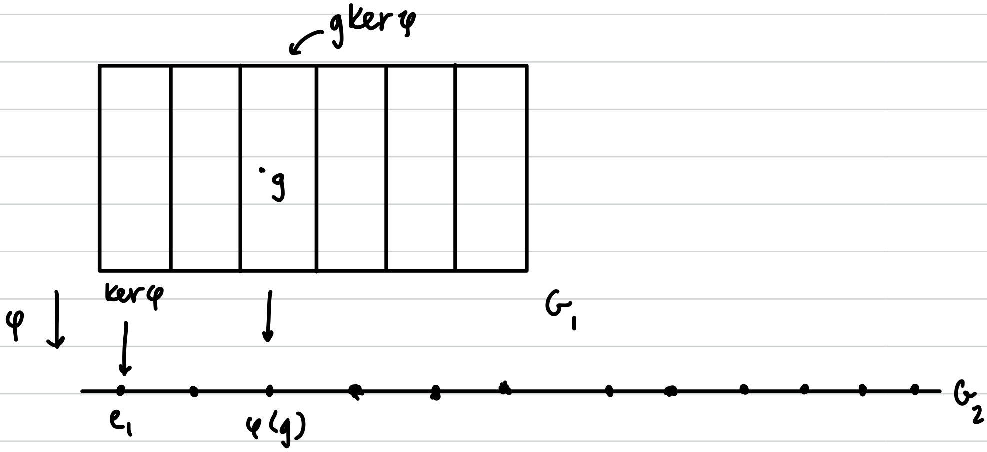
First Isomorphism Thm: Sps $\varphi: G_1 \to G_2$ homomorphism.



Then: $G_1/\ker\varphi \cong \varphi(G_1)$

via $\Phi(g\ker\varphi) = \varphi(g)$

Thm
$\phi: G_1 \to G_2$ hm
$G_1/\ker\varphi \cong \varphi(G_1)$
via $\Phi(g\ker\varphi) = \varphi(g)$

proof of thm:

because formula for Φ based on coset representative

NTS: Φ well-defined

and Φ an isomorphism

1-1, onto, operation-preserving

Φ is well-defined:

ie $\varphi(g) = \varphi(h)$

Sps $g\ker\varphi = h\ker\varphi$. (NTS: $\Phi(g\ker\varphi) = \Phi(h\ker\varphi)$)

litmus test

Then $h^{-1}g \in \ker\varphi$ so

$$e_2 = \varphi(h^{-1}g) = \varphi(h^{-1})\varphi(g) = [\varphi(h)]^{-1}\varphi(g)$$

✓

Thus $\varphi(g) = \varphi(h)$, so by definition of Φ, $\Phi(g\ker\varphi) = \Phi(h\ker\varphi)$.

Φ is 1-1:

Sps $\Phi(g\ker\varphi) = \Phi(h\ker\varphi)$. (NTS: $g\ker\varphi = h\ker\varphi$)

By defn of Φ, $\varphi(g) = \varphi(h)$

litmus test

Then $\varphi(h^{-1}g) = e_2$ so $h^{-1}g \in \ker\varphi$. Thus $g\ker\varphi = h\ker\varphi$.

Φ is onto $\varphi(G_1)$: (a subgroup of G_2, not necessarily onto G_2).

Sps $x \in \varphi(G_1)$. Then $x = \varphi(g)$ for some $g \in G_1$.

So $x = \varphi(g) = \Phi(g\ker\varphi)$ by definition of Φ. Thus Φ is onto.

Φ is operation-preserving:

$\Phi: G_1/\ker\varphi \to \varphi(G_1)$

$$\begin{aligned}
\Phi((g_1\ker\varphi)(g_2\ker\varphi)) &= \Phi(g_1 g_2 \ker\varphi) \\
&= \varphi(g_1 g_2) \quad \text{(by defn of } \Phi\text{)} \\
&= \varphi(g_1)\varphi(g_2) \quad \text{(since } \varphi \text{ is a homomorphism)} \\
&= \Phi(g_1\ker\varphi)\,\Phi(g_2\ker\varphi) \quad \text{(by defn of } \Phi\text{)}
\end{aligned}$$