rotations

Ex $D_n/R \approx \mathbb{Z}_2$ because: $\varphi : D_n \longrightarrow \mathbb{Z}_2$

Define $\varphi(s) = \begin{cases} 0 & s \text{ rotation} \\ 1 & s \text{ flip} \end{cases}$

φ a homomorphism. $\varphi(xy) = \varphi(x) + \varphi(y)$ $\quad x, y =$ flip or rotation

φ is onto. ✓

$\ker \varphi = R.$

↳ By first isomorphism theorem,

$$\underset{G_1}{D_n} / \underset{\ker\varphi}{R} \cong \underset{\varphi(G_1)}{\mathbb{Z}_2}$$

Ex. $\varphi : \mathbb{Z}_8 \to \mathbb{Z}_4$

$\varphi(x) = 2x \bmod 4$

$$\begin{array}{c|cccccccc} \mathbb{Z}_8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \mathbb{Z}_4 & 0 & 2 & 0 & 2 & 0 & 2 & 0 & 2 \end{array}$$

$\{0, 1, 2, 3\}$

Here, φ a homomorphism$^{\mathbb{Z}_8 \to \mathbb{Z}_4}$ (exercise)

$\ker \varphi = \langle 2 \rangle$ (in $\mathbb{Z}_8$)

$\varphi(\mathbb{Z}_8) = \underbrace{\langle 2 \rangle}_{\varphi(G_1)}$ (in $\mathbb{Z}_4$)

So, by first isomorphism theorem,

$$\underbrace{\mathbb{Z}_8 / \langle 2 \rangle}_{\text{in } \mathbb{Z}_8} \approx \underbrace{\langle 2 \rangle}_{\text{in } \mathbb{Z}_4}$$