Rings

↳ Rings are abelian groups with an additional binary operation.

Defn A ring is a nonempty set R along with two binary operations, addition and multiplication, such that for all $a, b, c \in R$,

abelian group under addition

1. there exists an element $0 \in R$ s.t. $0 + a = a + 0 = a$
2. For each $a \in R$ there exists an element $-a$ s.t. $a + (-a) = (-a) + a = 0$
3. $(a+b)+c = a+(b+c)$
4. $a+b = b+a$
5. $a(bc) = (ab)c$
6. $a(b+c) = ab+ac$ and $(a+b)c = ac+bc$

distribution

Notes:

- Part of the definition is closure under addition and multiplication.

- Properties 1-4 say R is an abelian group under addition.

- Property 6 says addition and multiplication interact well.

- R is not necessarily a group under multiplication
  - ↳ might not have multiplicative identity
  - ↳ might not have multiplicative inverses.

- Similar to groups, rings are meant to abstract properties of familiar number systems.
  - ↳ addition (inverse = subtraction)
  - ↳ multiplication (inverse = division)