<u>Ex</u> $\mathbb{Z}$

abelian group under addition? ✓

closed under multiplication? ✓

$a(b+c) = ab + ac$ ✓

$(a+b)c = ac + bc$ ✓

$a(bc) = (ab)c$ ✓

$\mathbb{Z}$ is a commutative ring: $ab = ba$ for all $a, b \in \mathbb{Z}$.

It's also a ring with unity (multiplicative identity): $\underline{1}$

$1n = n1 = n$ for all $n \in \mathbb{Z}$.

<u>Ex</u> $\mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\}$

abelian group under addition mod n

closed under multiplication mod n

Properties of modular arithmetic

$a(b+c) = ab + ac \qquad (a+b)(c) = ac + bc$

$a(bc) = (ab)c.$

Commutative ring? yes.

Ring with unity? yes: 1 = unity

<u>Ex</u> $5\mathbb{Z} = \{\ldots, -20, -15, -10, -5, 0, 5, 10, 15, \ldots\}$

↳ commutative ring <u>without</u> unity

4. $M_2(\mathbb{Z}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \,\middle|\, a,b,c,d \in \mathbb{Z} \right\}$

Addition: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

in $\mathbb{Z}$ b/c $\mathbb{Z}$ ring

Multiplication: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & * \\ * & * \end{bmatrix}$

$\downarrow$

matrix mult.

Since $\mathbb{Z}$ is closed under addition and multiplication, $M_2(\mathbb{Z})$ is closed under matrix multiplication.

Commutative? No.

Note: 1 is mult. identity in $\mathbb{Z}$

Unity? yes: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

In fact,

closed under matrix multiplication for the same reason as above

5. If R is any ring, so is $M_2(R)$.

If R has unity, so does $M_2(R)$.