6. $\mathbb{Q}$ = rationals = $\{ \frac{a}{b} \mid a, b \in \mathbb{Z},\ b \neq 0 \}$ $\qquad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

commutative? yes.

unity? 1

contrast with $\mathbb{Z}$

* multiplicative inverses? yes.

for all nonzero elements

7. $\mathbb{R}$ = real numbers

$(i = \sqrt{-1})$

adjoin i

8. $\mathbb{C} = \mathbb{R}[i] = \{ a + bi \mid a, b \in \mathbb{R},\ i^2 = -1 \}$

$$(a+bi)(c+di) = ac + bci + adi + bd\underbrace{i^2}_{-1}$$
$$= (ac - bd) + (bc + ad)i$$

commutative, unity, multiplicative inverses. ✓

for all nonzero elements

9. $\mathbb{Z}[i] = \{a+bi \mid a, b \in \mathbb{Z},\ i^2 = -1\}$

same multiplication as $\mathbb{C}$.

↳ commutative, with unity.

10. Sps R is a ring. The <u>polynomial ring over R</u> is

$$R[x] = \{a_0 + a_1 x + \cdots + a_n x^n \mid a_i \in R\}$$

e.g. $1 + 2x - 9x^3 \in \mathbb{Z}[x]$.

Ring operations: polynomial addition and multiplication.

e.g. $(1+2x) + (3x + 7x^2) = 1 + 5x + 7x^2$

$$(1+2x)(3x+7x^2) = 3x + 7x^2 + 6x^2 + 14x^3$$
$$= 3x + 13x^2 + 14x^3$$

Note: elements in $R[x]$ are just formal polynomials. Generally, we think of x as a placeholder that helps us define addition and multiplication, rather than as a variable to plug into.

11. If R_1 and R_2 are rings, can form the direct product

$R_1 \oplus R_2$.

$\hookrightarrow \{(r_1, r_2) \mid r_1 \in R_1, r_2 \in R_2\}$

componentwise operations.