Defn If R is a ring with unity and $a \in R$ has a multiplicative inverse a^{-1}, we say a is a **unit** in R.

Ex In $\mathbb{Z}_6 = \{0,1,2,3,4,5\}$

5 is a unit $\rightsquigarrow$ $5^{-1} = 5$

4 is not a unit:

$4 \cdot 0 = 0$	$4 \cdot 3 = 0$
$4 \cdot 1 = 4$	$4 \cdot 4 = 4$
$4 \cdot 2 = 2$	$4 \cdot 5 = 2$

Defn In R, if $a \neq 0$, and if there exists b such that $ab = c$, we say a **divides** c, denoted $a \mid c$.

just like in $\mathbb{Z}$.

(avoid writing: $b = \frac{c}{a}$.)

(doesn't have meaning in many rings.

<u>Ex</u> In $\mathbb{Z}_6$,

$$4 \mid 2 \text{ because } \overset{a}{4} \cdot \overset{b}{2} = \overset{c}{2}.$$

* ----- also $4 \cdot 5 = 2$

Note: if $ax = c$ has a solution, it is often not unique.

OTOH, $4 \nmid 1$.

So:

WARNING: unlike groups, in general in a ring, $ab = ac \not\Rightarrow b = c$.

In $\mathbb{Z}_6$ $4 \cdot 2 = 4 \cdot 5$ but $2 \neq 5$.