Properties of rings

Suppose R is a ring.

additive identity

1. $a0 = 0a = 0$ for all $a \in R$.

$a0 = a(0+0) = a0 + a0$ (subtract $a0$ both sides)

$\Rightarrow 0 = a0$

additive inverse of b additive inverse of ab

2. $a(-b) = (-a)b = -(ab)$

additive inverse of a

$a(-b) + ab = a(-b+b)$

$= a0$

$= 0$

so $a(-b) = -(ab)$. Similar for $(-a)b$.

3. $(-a)(-b) = ab$

4. $a(b-c) = ab - ac$, ie.

$$a(b+(-c)) = ab + a(-c) = ab + (-(ac)) = ab - ac.$$

additive inverse of 1

5. If R is a ring with unity, $(-1)a = -a$

additive inverse of a.

(follows from property 2)

6. $(-1)(-1) = 1$

7. If R has unity, it is unique.

↳ same proof as for groups. (exercise)

8. If R has unity and if $a \in R$ has a multiplicative inverse a^{-1}, then a^{-1} is the unique inverse of a.

↳ same proof as for groups. (exercise)

$a^n \rightsquigarrow na$

Notation: Recall additive notation in groups:

$$\underbrace{a + a + \cdots + a}_{n \text{ times}} = na$$

Can be ambiguous now, so write

$$\underbrace{a + a + \cdots + a}_{n \text{ times}} = n \cdot a.$$