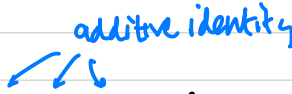


## Properties of rings

Suppose  $R$  is a ring.

1.  $a0 = 0a = 0$  for all  $a \in R$ .

additive identity



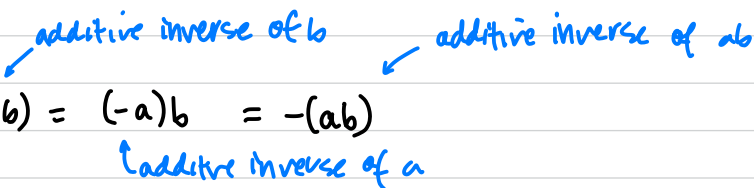
$$a0 = a(0+0) = a0 + a0 \quad (\text{subtract } a0 \text{ both sides})$$

$$\Rightarrow 0 = a0$$

2.  $a(-b) = (-a)b = -(ab)$

additive inverse of  $b$       additive inverse of  $ab$

additive inverse of  $a$




$$a(-b) + ab = a(-b + b)$$

$$= a0$$

$$= 0$$

so  $a(-b) = -(ab)$ . Similar for  $(-a)b$ .



$$3. (-a)(-b) = ab$$

$$4. a(b-c) = ab - ac, \text{ i.e.}$$

$$a(b+(-c)) = ab + a(-c) = ab + (-ac) = ab - ac.$$

$$5. \text{ If } R \text{ is a ring with unity, } (-1)a = -a$$

↙ additive inverse of 1

↑ additive inverse of a.

(follows from property 2)

$$6. (-1)(-1) = 1$$

7. If  $R$  has unity, it is unique.

↳ same proof as for groups. (exercise)

8. If  $R$  has unity and  $a \in R$  has a multiplicative inverse  $a^{-1}$ ,  
then  $a^{-1}$  is the unique inverse of  $a$ .

↳ same proof as for groups. (exercise)

$$a^n \rightsquigarrow na$$

Notation: Recall additive notation in groups:

$$\underbrace{a+a+\dots+a}_{n \text{ times}} = na$$

Can be ambiguous how, so write

$$\underbrace{a+a+\dots+a}_{n \text{ times}} = n \cdot a.$$