**Defn** A subset $S \subseteq R$ ring is a <u>subring</u> if $S$ is a ring under the same binary operations as $R$.

**Ex.** $5\mathbb{Z} \subset \mathbb{Z}$ is a subring.

**Nonex.** Sps $R$ a ring. Let $S = \{a_0 + a_1 x \mid a_0, a_1 \in R\}$.

$S$ is not a subring of $R[x]$.

Why? Not closed under multiplication:

$$(a_0 + a_1 x)(b_0 + b_1 x) = a_0 b_0 + (a_1 b_0 + a_0 b_1)x + a_1 b_1 x^2$$

if $a_1 b_1 \neq 0$, product not in $S$.

go back and review defn of ring

<u>Thm</u> (Subring Test)

Sps $S \subseteq R$ is not empty. If $a-b \in S$ and $ab \in S$ for all $a,b \in S$, then $S$ is a subring of $R$.

based on onestep subgroup test.

proof: Since $a-b \in S$ for all $a, b \in S$, $S$ is an (abelian) subgroup

Since $ab \in S$, $S$ is closed under multiplication.

Associativity and distribution hold because they hold in $R$.

**Defn** Sps $a \in R$ is nonzero. We say $a$ is a **zero divisor** if there exists $b \neq 0$ in $R$ such that $ab = 0$.

**Ex** zero divisors in $\mathbb{Z}_6 = \{0,1,2,3,4,5\}$ : $2, 3, 4$

ex $4 \cdot 2 = 4 \cdot 5$ in $\mathbb{Z}_6$

Note! zero divisors mess up cancellation:

Recall that in rings, $ab = ac \not\Rightarrow b = c$

this is because it could be that $a(b-c) = 0$ even if $b \neq c$.

$4 \cdot (5 - 2) = 4 \cdot 3 = 0$ in $\mathbb{Z}_6$