<u>Defn</u> The <u>characteristic</u> of a ring R, denoted char R, is the smallest positive integer n such that $n \cdot x = 0$ for all $x \in R$.

If no such integer exists, we say char $R = 0$

Recall: $n \cdot x = \underbrace{x + x + \cdots + x}_{n \text{ times}}$

Note: Sps $|R|$ finite.

char R will be the lcm of the additive orders of the elements of R.

Since the order of each element divides $|R|$, it follows that char R divides $|R|$.

Ex char $\mathbb{Z}_n = n$.

will be $\leq n$. But $|1| = n$, so char $R \geq n$

Ex char $\mathbb{C} = 0$

Ex char $M_2(\mathbb{Z}_4) = 4$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $a, b, c, d \in \mathbb{Z}_4$

Ex Sps R is a commutative and char $R = 2$.

Then $(a+b)^2 = a^2 + b^2$.

Indeed,

$(a+b)^2 = (a- \quad)(a+b)$

char $R = 0$

$= a^2 + ba + ab + b^2 = a^2 + 2ab + b^2$

$= a^2 + b^2$. ✓