<u>Thm</u> Sps R has unity 1. If additive order of 1 is infinite, char $R = 0$. If additive order of 1 is n, then char $R = n$.

↑ i.e. can find char by considering 1.

proof: First statement follows from defn of char R.

OTOH, sps $|1| = n$. Then, by defn, char $R \geq n$.

Sps. $x \in R$. Then

$$
\begin{aligned}
n \cdot x &= x + \cdots + x \\
&= (1x + \cdots + 1x) \\
&= (1 + \cdots + 1)x \\
&= (n \cdot 1)x \\
&= 0x = 0
\end{aligned}
$$

Since $n \cdot x = 0$ for all $x \in R$, char $R \leq n$. So char $R = n$.

Thm If R is an integral domain, char R is 0 or prime.

proof: Recall that by defn of integral domain, R has unity.

Sps. char R not 0. (NTS: char R is prime.)

For contradiction, sps. char $R = n$, not prime, so $n = xy$

By our previous theorem, n is the additive order of 1, so is the smallest n such that $n \cdot 1 = 0$.

But then

exercise

$$0 = n \cdot 1 = (x \cdot 1)(y \cdot 1)$$

neither is 0

Since R is an integral domain, this says $x \cdot 1 = 0$ or $y \cdot 1 = 0$, a contradiction.

Thus char R is prime.