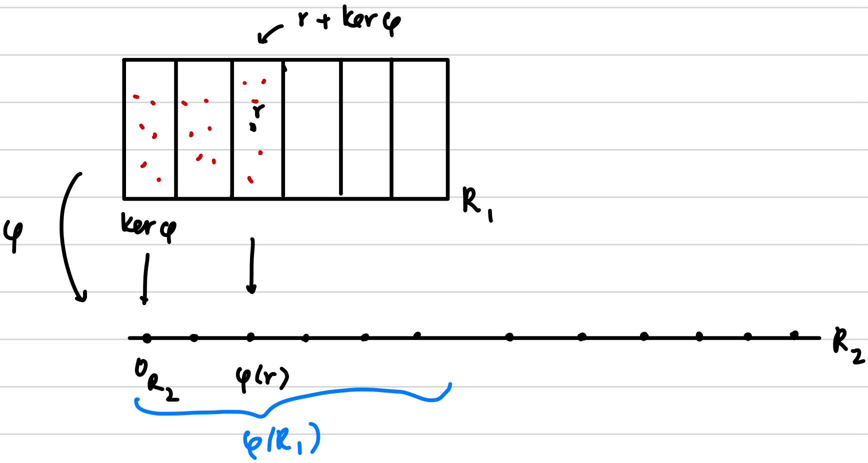
Sps $\varphi : R_1 \longrightarrow R_2$ a ring homomorphism.



additive cosets: $\{r + \ker\varphi \mid r \in R_1\}$ $\rightarrow$ $R_1 / \ker\varphi \cong \varphi(R_1)$

via $\overline{\Phi}(r + \ker\varphi) = \varphi(r)$

proof: By first isomorphism theorem for groups, we know

$$R_1 / \ker\varphi \cong \varphi(R_1) \text{ as additive groups.}$$

(So we've done: $\overline{\Phi}$ well-defined, 1-1, onto, addition preserving)

Only need to check that $\overline{\Phi}$ preserves multiplication.

But

$$\begin{aligned}
\overline{\Phi}\left((r_1+\ker\varphi)(r_2+\ker\varphi)\right) &= \overline{\Phi}(r_1 r_2 + \ker\varphi) \\
&= \varphi(r_1 r_2) \\
&= \varphi(r_1)\varphi(r_2) \\
&= \overline{\Phi}(r_1+\ker\varphi)\,\overline{\Phi}(r_2+\ker\varphi) \quad \checkmark
\end{aligned}$$

So $\overline{\Phi}$ preserves coset multiplication