$\varphi: R_1 \to R_2$

$R_1/\ker\varphi \cong \varphi(R_1)$

<u>Ex</u> $\underbrace{\mathbb{Z}/\underbrace{n\mathbb{Z}}_{\ker\varphi}}_{R_1} \cong \underbrace{\mathbb{Z}_n}_{R_2}$, as rings

$\varphi: \mathbb{Z} \to \mathbb{Z}_n$

$\varphi(x) = x \bmod n \rightsquigarrow \varphi$ ring homomorphism ✓

onto ✓

$\ker\varphi = n\mathbb{Z}$. ✓

see earlier notes

<u>Ex</u> $\underbrace{\mathbb{R}[x]}_{R_1}/\underbrace{\langle x\rangle}_{\ker\varphi} \cong \underbrace{\mathbb{R}}_{R_2}$ as rings.

Consider $\varphi: \mathbb{R}[x] \to \mathbb{R}$ given by

$\varphi(f(x)) = f(0)$ ⟵ evaluation map, a ring homomorphism

constant term of polynomial f.

$\ker\varphi = \{\text{polynomials } f(x) \text{ with constant term } 0\} = \langle x\rangle$

φ onto? Yes. For example, if $r \in \mathbb{R}$, then $r = \varphi(r + 2x)$, for example.

So, by first isomorphism theorem for rings: $\mathbb{R}[x]/\langle x\rangle \cong \mathbb{R}$.