

of ring homomorphism

← compare with normal subgroups of groups.

Finally, we know kernels are ideals.

Turns out, ideals are kernels too:

Thm Sp. $I \subset R$ is an ideal. Then I is the kernel of a ring homomorphism.

proof: Consider $\varphi: R \rightarrow R/I$ given by $\varphi(r) = r + I$.

↳ check: φ is a ring homomorphism (exercise)

The additive identity in R/I is I (a.k.a. $0 + I$), so

$$r \in \ker \varphi \Leftrightarrow \varphi(r) = I \Leftrightarrow r + I = I \Leftrightarrow r \in I.$$

Thus $I = \ker \varphi$, as desired.