Groups

"maps to"

$(g_1, g_2) \mapsto g_1 g_2.$

a.k.a. function

Defn For a set G, a map $G \times G \to G$ is called a binary operation.

$G \times G = \{(g_1, g_2) \mid g_1, g_2 \in G\}$

e.g. $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
$= \{(x,y) \mid x, y \in \mathbb{R}\}$

Defn A nonempty set G together with a binary operation $(a,b) \mapsto ab$ is a group if

1. $(ab)c = a(bc)$ for all $a, b, c \in G$. (associativity)
2. there exists an element $e \in G$ s.t.

   $ae = ea = a$ for all $a \in G$.

   (e is the identity.)

3. for each $a \in G$ there is an element $a^{-1} \in G$ s.t.

   $aa^{-1} = a^{-1}a = e.$

   (each $a \in G$ has an inverse.)

(Note: closure under binary operation is part of the defn.)

4 things to check:
1. closed under operation
2. associativity
3. identity
4. inverses.

If $ab = ba$ for all $a, b \in G$, G is called abelian.

Notes:

1. you need both a set and an operation to define a group.

2. generically, we call the group operation "multiplication".
(e.g. ab is "a times b" or "a mult. by b".