More examples of groups

$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

general linear

"A"

7. $GL(2,\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \,\middle|\, a,b,c,d \in \mathbb{R},\ \det A \neq 0 \right\}$

nonabelian (exercise)

under matrix multiplication.

Notice: $I = AA^{-1}$

so $\det I = \det A \det A^{-1}$

so $1 = \det A \det A^{-1} \Rightarrow \det A^{-1} = \frac{1}{\det A}$. *

Fact: $\det AB = \det A \det B$

Sps $A, B \in GL(2,\mathbb{R})$

1. closed? product is a 2×2 real matrix ✓

Also: $\det AB = \det A \det B$ Thus $AB \in GL(2,\mathbb{R})$ ✓

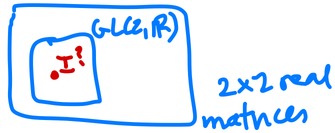
neither is 0, so $\det AB \neq 0$.

2. Associativity: b/c function composition is assoc, and b/c 2×2 matrices represent L.T.'s (ie functions) matrix mult is associative.

linear transformations

represents outcome of composition

3. Identity? Consider $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Notice $IA = AI = A$ for all $A \in GL(2,\mathbb{R})$



Check that $I \in GL(2,\mathbb{R})$.

$\hookrightarrow$ I is 2x2, real matrix and $\det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \neq 0$. ✓

4. Inverses? Sps $A \in GL(2,\mathbb{R})$. Since $\det A \neq 0$, A is invertible.

Q: Is $A^{-1} \in GL(2,\mathbb{R})$?

ie. $\exists A^{-1}$ s.t. $AA^{-1} = A^{-1}A = I$. It's 2x2, real. ✓ By *, $\det A^{-1} \neq 0$. ✓

So $A^{-1} \in GL(2,\mathbb{R})$.

8. $M_{2\times 2}(\mathbb{Z}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, a,b,c,d \in \mathbb{Z} \right\}$

under matrix addition.

$\hookrightarrow$ exercise.

9. Symmetries (rotations) of a circle. (abelian)

orientation-preserving

column vectors, $\mathbb{R}$ entries

10. $\mathbb{R}^n$ under vector addition (abelian)

11. $\mathbb{R}^* = \{ x \in \mathbb{R} \mid x \neq 0 \} = \mathbb{R} - \{0\}$ under multiplication.

compare with $U(n)$ when n prime

12. $\{1, 2, 3, \ldots, n-1\}$ under multiplication modulo n is not a group if n is not prime. Why? not closed under binary when $n=12$. Ex: 3,4