Goal: understand groups in general.

<u>Thm</u> Let G be a group. The identity element in G is unique.

proof: Sps. e_1 and e_2 are both identity elements in G.

(NTS: $e_1 = e_2$.)

If e_1 and e_2 are both identity elements, then

$$e_1 a = a \quad \forall a \in G \qquad (1)$$

$$a e_2 = a \quad \forall a \in G. \qquad (2)$$

Thus

$$\underbrace{e_1 = e_1 e_2}_{\substack{\text{equ (2) with} \\ a = e_1}} \underbrace{= e_2}_{\substack{\text{eqn (1)} \\ \text{with } a = e_2.}}. \checkmark$$

* Note: usually denote the identity elt. by e.

Thm For all a, b, c in a group G,

$$ab = ac \Rightarrow b = c.$$

So: cancellation holds in a group.

proof: Since $a \in G$, $\exists\, a^{-1} \in G$ s.t. $a^{-1}a = e$.

Therefore

$$ab = ac$$

$$\Rightarrow \underbrace{a^{-1}a}_{e}b = \underbrace{a^{-1}a}_{e}c$$

$$\Rightarrow eb = ec \quad \Rightarrow \quad b = c. \checkmark$$

mult on left* both sides by a^{-1}

Similarly, $ba = ca \Rightarrow b = c$

why? $ba = ca$

$$\Rightarrow baa^{-1} = caa^{-1}$$

$$\Rightarrow b = c.$$

mult on right* bothsides by a^{-1}.

But CAREFUL: In general, $ab = ca \not\Rightarrow b = c$.

↳ multiplying on left is not same as multiplying on right in general.

think of x as variable... can solve for x.

Ex $axc = b \iff x = a^{-1}bc^{-1}$

$$axc = b$$

$$\iff \underbrace{a^{-1}a}_{e}xc = a^{-1}b$$

$$\iff xc = a^{-1}b$$

$$\iff x\underbrace{cc^{-1}}_{e} = a^{-1}bc^{-1}$$

$$\iff x = a^{-1}bc^{-1}.$$

Recall: in our group table (a.k.a. Cayley table) for D_3, each element appeared exactly once in each row and column.

Why?

right →

left ↓

	g_1	g_2	$\cdots$	g_n
g_1	$g_1 g_1$	$g_1 g_2$		$g_1 g_n$
g_2				
$\vdots$				
g_n	$g_n g_1$			

each group element listed one time.

proof by contradiction

Sps. g shows up twice in the i^{th} row.

This implies $g_i g_j = g_i g_k$ for some g_j, g_k.

But by cancellation, $g_j = g_k$. Contradicts fact that each gp. elt shows up only once across top of group table. Thus g shows up only once in i^{th} row. Similar for columns.