

Defn The number of elements in a group  $G$  is the order of  $G$ , denoted  $|G|$ .

$$= \{1, 3, 7, 9\}$$

Ex  $|U(10)| = 4$ .

$GL(2, \mathbb{R})$  has infinite order.

Defn The order of an element  $g$  in a group  $G$ , denoted  $|g|$ , is the smallest positive integer  $n$  s.t.  $g^n = e$ .

If there is no such  $n$ , we say  $g$  has infinite order.

Ex. In  $\mathbb{Z}_{15}$ ,  $|5| = 3$

$|12| = 15$

$\mathbb{Z}_5 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

(Note: since  $\mathbb{Z}_5$  is additive, we write  $3 \cdot 5 = 0$  instead of  $5^3 = 0$ .)

Ex In  $\mathbb{Q}^*$ ,  $\frac{1}{3}$  has infinite order.

↳  $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \dots$