Subgroups

Consider $D_3 = \{R_0, R_{120}, R_{240}, F_1, F_2, F_3\}$.

↳ in this group, the subset $\{R_0, R_{120}, R_{240}\}$ by itself satisfies the defn. of a group.

$\{R_0, R_{120}, R_{240}\}$ is a subgroup of D_3.

Defn A subset H of a group G that satisfies the definition of a group under the operation of G is called a subgroup of G. Denoted $H \leq G$ (or $H < G$ if H is strictly smaller than G).

↳ if $H < G$, say H is a proper subgroup of G.

Ex. Subgroup of $\mathbb{Z}$: $\{\text{evens}\} = \{ \ldots, -4, -2, 0, 2, 4, 6 \ldots \}$

addition

Nonex: $\{\text{odds}\} \subseteq \mathbb{Z}$ is not a subgp b/c not closed under addn.

Nonex $\underbrace{\mathbb{Z}_n}_{\{0,1,2,\ldots,n-1\}} \not\leq \mathbb{Z}$ for any n.

↳ why? operations aren't same.

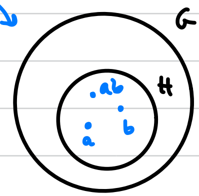
(addn mod n vs. addition)

Note: Every group has the trivial subgroup, i.e. $H = \{e\}$.

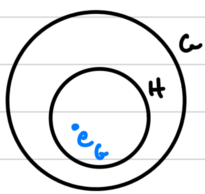
Note: Because associativity holds in G, and H inherits its operation from G, when checking whether H is a subgp, no need to check that associativity holds.

Thus, at the most basic level, there are three steps to proving that a nonempty subset H of a group G is a subgroup:

1. if $a, b \in H$, then $ab \in H$ (H closed under binary operation)



2. $e_G \in H$.



3. if $a \in H$, then $a^{-1} \in H$.

   (H closed under inverses).