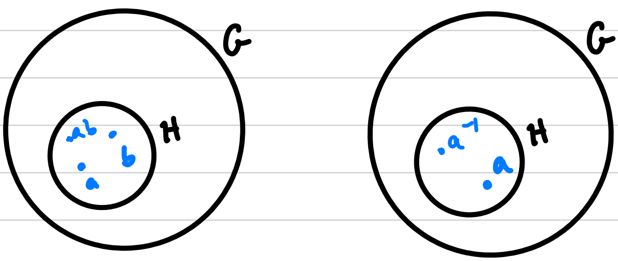
There are a couple of shorter ways to check that H is a subgroup:

Thm (Two-Step Subgroup Test)

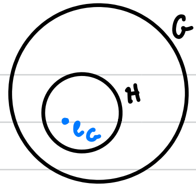
Sps H is a nonempty subset of a group G.

If H is closed under the binary operation and if $a^{-1} \in H$ whenever $a \in H$, then H is a subgroup of G.



Remark: This theorem tells us that you get the identity in H for free if you have closure and inverses.

proof: We only need to show $e_G \in H$.



Since H is nonempty, there is some element $a \in H$.

But then, by hypothesis, $a^{-1} \in H$.

Since H is closed under the binary operation,

$$aa^{-1} \in H, \quad \text{ie. } e_G \in H. \checkmark$$

both in H, by above

Ex. Let $G = GL(2, \mathbb{R})$ and let

$\det A \neq 0$

$$H = SL(2,\mathbb{R}) = \{ A \in GL(2,\mathbb{R}) \mid \det A = 1 \}$$

special linear

subset

Note: $H \subseteq G$

subgroup

$H \leq G$ as follows:

- H nonempty? yes.
- Sps $A, B \in H$. Then $\det A = \det B = 1$.

  So $\det AB = \det A \det B = 1 \cdot 1 = 1$ exists in $GL(2,\mathbb{R})$.

  so: closed under binary operation.
- Sps $A \in H$, so $\det A = 1$. Consider A^{-1}.

  $\det A^{-1} = \dfrac{1}{\det A} = \dfrac{1}{1} = 1$. so: closed under inverses. $\checkmark$