

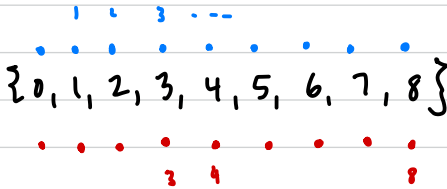
# Cyclic Groups

Defn A group  $G$  is called cyclic if there is some  $a \in G$  s.t.  $G = \langle a \rangle$ . In this case,  $a$  is called a generator of  $G$ .

↳ recall:  
 $\langle a \rangle = \{ a^i \mid i \in \mathbb{Z} \}$

Ex.  $\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\} = \langle 1 \rangle$  for any  $n$ .  
↳ 1 is a generator

Ex.  $\mathbb{Z}_9 = \langle 1 \rangle = \langle 4 \rangle$  ↳ multiple generators for cyclic groups



Note: cyclic groups are abelian b/c

for all  $k, m \in \mathbb{Z}$

$$a^k a^m = a^{k+m} = a^{m+k} = a^m a^k$$

↳  
generators of  $\langle a \rangle$  ... and they commute.

Nonex  $D_3$  is not cyclic.

↳  $D_3$  not abelian, so can't be cyclic

↳ alt, direct path: compute  $\langle x \rangle$  for all  $x \in D_3$

↳ none equal to  $D_3$ .