

$\langle a \rangle$

Thm Sps  $G$  is a group and  $a \in G$ .

1. If  $a$  has infinite order, then  $a^i = a^j \Leftrightarrow i = j$ .

2. If  $|a| = n$ , then

a.  $a^i = a^j \Leftrightarrow n | i - j$ .

b.  $\langle a \rangle = \{e, a^1, a^2, \dots, a^{n-1}\}$  (Note similarity w/ $\mathbb{Z}_n$ .)

idea: cycle through group elements (repeatedly if  $|a|$  is finite)

Corollary  $|a| = |\langle a \rangle|$

order of  $a$  as an element  $\uparrow$  # elts in set  $\langle a \rangle$ .

\* Corollary Let  $G$  be a group and  $a \in G$ . Then  $a^k = e$  if and

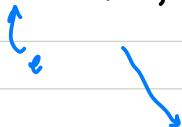
only if  $|a| \mid k$ .

says  $|a|$  divides  $k$  but  
not necessarily  $|a|=k$

proof: From (2a) in thm,  $a^k = a^0 = e \Leftrightarrow |a| \mid k - 0$ , i.e.  $|a| \mid k$ .

Ex In some group  $G$ , if  $x^8 = e$ , possibilities for

$|x|$  are: 1, 2, 4, or 8.



sps  $x^2 = e \dots$

$$x^8 = (x^2)^4 = e^4 = e.$$