$\langle a \rangle$

Thm Sps G is a group and $a \in G$.

1. If a has infinite order, then $a^i = a^j \Leftrightarrow i = j$.

i.e. a has finite order

2. If $|a| = n$, then

   a. $a^i = a^j \Leftrightarrow n \mid i - j$.

   b. $\langle a \rangle = \{\overset{a^0}{e}, a^1, a^2, \ldots, a^{n-1}\}$ (Note similarity w/ $\mathbb{Z}_n$.)

idea: cycle through group elements (repeatedly if $|a|$ is finite)

Corollary $\underbrace{|a|}_{\text{order of } a \text{ as an element}} = \underbrace{|\langle a \rangle|}_{\#\text{ elts in set } \langle a \rangle}$

Corollary Let G be a group and $a \in G$. Then $a^k = e$ if and only if $|a| \mid k$.

$\leftarrow$ says $|a|$ divides k but not necessarily $|a| = k$

proof: From (2a) in thm, $a^k = a^0 = e \Leftrightarrow |a| \mid k - 0$, i.e. $|a| \mid k$.

Ex In some group G, if $x^8 = e$, possibilities for

$|x|$ are: 1, 2, 4, or 8.

e

sps $x^2 = e \ldots$

$x^8 = (x^2)^4 = e^4 = e.$