From now on, assume A is finite. Label the elements of A as $\{1, 2, 3, \dots, n\}$

these are the things being permuted, not the permutations.

S_n denotes the permutation group of A

Note: $|S_n| = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1 = n!$

Notation / Example

$$S_4 = \{\underbrace{\text{permutations}}_{\text{bijections}} \text{ of } \{1,2,3,4\}\}$$

bijections

Ex $\alpha, \beta \in S_4$

$\alpha(1) = 4 \quad \alpha(2) = 3 \quad \alpha(3) = 2 \quad \alpha(4) = 1$

$\beta(1) = 2 \quad \beta(2) = 3 \quad \beta(3) = 1 \quad \beta(4) = 4$

a permutation

e.g. $(\alpha\beta)(2) = \alpha(\beta(2)) = \alpha(3) = 2.$

not same

$(\beta\alpha)(2) = \beta(\alpha(2)) = \beta(3) = 1$

a permutation

Note: since $(\alpha\beta)(2) \neq \overset{(\beta\alpha)(2)}{\cancel{\beta(\alpha(2))}}$, we conclude that

$\alpha\beta \neq \beta\alpha$ as elements of S_4. So S_4 is not abelian.

not equal as permutations (i.e. functions)