

Cycle Notation

Sps α maps

x	1	2	3	4	5	6	7
	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
$\alpha(x)$	6	3	4	7	5	1	2

Denote this in (disjoint) cycle notation by:

$$\underbrace{(1\ 6)}_{2\text{-cycle}} \underbrace{(2\ 3\ 4\ 7)}_{4\text{-cycle}} \in S_7$$

\uparrow a.k.a. transposition

Notes:

1. A 2-cycle is also called a transposition.

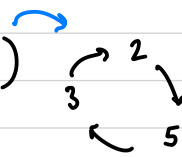
2. ~~***~~ Usually don't write 1-cycles:

$$(1\ 6)(2\ 3\ 4\ 7) = (1\ 6)(2\ 3\ 4\ 7)(5)$$

\hookrightarrow if a number doesn't appear in a cycle, it's understood that the number maps to itself.

* Have to keep track of which S_n you are considering.

* * Exception: $(1) = (2) = (3) = \dots$

3. $(2\ 5\ 3) = (5\ 3\ 2) = (3\ 2\ 5)$  $\neq (3\ 5\ 2)$

4. Disjoint cycles commute:

$$\underline{(1\ 6)}(2\ 3\ 4\ 7) = (2\ 3\ 4\ 7)(1\ 6)$$

disjoint b/c no number appears in both cycles.

5. Every permutation can be expressed (using the algorithm above) as a cycle or a disjoint product of cycles.