Thm (Cayley's Thm) Every group G is isomorphic to a group of permutations.

2. Now, to see $G \approx \bar{G}$, define

$$\varphi : G \to \underbrace{\bar{G}}_{\{L_g \mid g \in G\}}$$

by $\varphi(g) = L_g$

WTS: $g=h$

Then φ is 1-1 because if $\varphi(g) = \varphi(h)$, ie. $L_g = L_h$ as functions.

So $L_g(e) = L_h(e)$. I.e. $ge = he$, ie. $g=h$. ✓

defn of L_g, L_h

Further, φ is onto because of how $\bar{G}$ is defined. ✓

Finally, to see that φ is operation-preserving :

$$\varphi(gh) = L_{gh} = L_g L_h$$

group operation in G

by above *

group operation in $\bar{G}$

Thus, $G \approx \bar{G}$, as desired.