

Cosets

Notation: Sp. H is a subset of a group G . Let $a \in G$. ↙ fixed

$$aH = \{ah \mid h \in H\} \quad \text{↖ subset of } G.$$

$$Ha = \{ha \mid h \in H\}$$

If H is a subgroup, we refer to aH (resp. Ha) ↙ "respectively"

as the left (resp. right) coset of H containing a .

$|aH|$ denotes the size of the set aH .

EX. $G = \mathbb{Z}$. $H = \{\text{evens}\} = \{\dots, -4, -2, 0, 2, 4, \dots\} = \langle 2 \rangle$

aH in additive notation

$a \rightarrow 1+H = \{\dots, -3, -1, 1, 3, 5, \dots\} = \{\text{odds}\}$

note: $1+H$ not a subgroup.

$$3+H = \{\dots, -1, 1, 3, 5, 7, \dots\} = 1+H \text{ as sets.}$$

$$2+H = \{\dots, -2, 0, 2, 4, 6, \dots\} = H \text{ as sets.}$$

We see: for a given coset, there is usually more than one way to represent it.