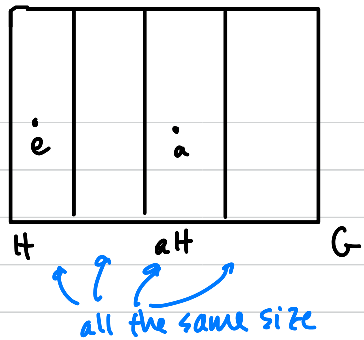
5. $|aH| = |bH|$ (even if they don't intersect)



Define $\varphi : aH \to bH$

$$ah \mapsto bh$$

"maps to"

Check : φ a bijection (i.e. 1-1, onto). (exercise).

6. $aH = Ha \iff aHa^{-1} = H$

$\{aha^{-1} \mid h \in H\}$.

Sps $aH = Ha$. Then $aH \subset Ha$.

not necessarily equal

This says that for all $h \in H$, there exists $h_1 \in H$ s.t.

$$ah = h_1 a.$$

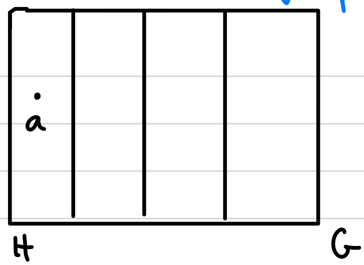
But this is equivalent to saying that for all $h \in H$,

there exists $h_1 \in H$ s.t.

$$aha^{-1} = h_1 \rightsquigarrow \text{ i.e. } aHa^{-1} \subset H.$$

i.e. $aH \subset Ha \iff aHa^{-1} \subset H$.

Now, show $Ha \subset aH \iff H \subset aHa^{-1}$ (exercise) the only coset that is a subgroup



7. aH a subgroup $\Leftrightarrow a \in H$.

property 3

Know $aH = H \Leftrightarrow a \in H$.

So: $a \in H \Rightarrow aH = H \Rightarrow aH$ a subgp. because H a subgp.

$a \notin H \Rightarrow aH \cap H = \emptyset \Rightarrow e \notin aH \Rightarrow aH$ not a subgroup of G.

property 2