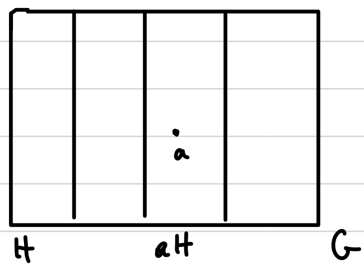
Thm (Lagrange's Thm)

If G is finite and $H \leq G$, then $|H|$ divides $|G|$.

The number of distinct left (or right) cosets of H in G

is $\dfrac{|G|}{|H|}$.



proof: By properties of cosets,

G is partitioned by the

distinct cosets of H:

$$G = a_1H \cup a_2H \cup \cdots \cup a_nH \qquad (a_iH \cap a_jH = \phi \text{ if } i \neq j)$$

Furthermore, $|a_iH| = |H|$ for all i.

So $\quad |G| = n|H|$. $\qquad$ (n = number of distinct cosets.)

so $|H| \,\big|\, |G|$. $\qquad$ and $n = \dfrac{|G|}{|H|}$.

Defn The **index** of H in G, denoted $[G:H]$, is the number of distinct cosets of H in G.

Note: $[G:H] = |G|/|H|$.

order of element divides order of group.

Corollary If $a \in G$, $|a|$ divides $|G|$.

proof: $|a| = |\langle a \rangle|$ and $\langle a \rangle \leq G$.

order of subgroup generated by a.

Corollary If $a \in G$, $a^{|G|} = e$.

So $G \cong \mathbb{Z}_p$.

Corollary If $|G| = p$, a prime, then G is cyclic.

proof: Sps $a \in G$ and $a \neq e$.

Since $|a|$ divides p, $|a| = p$, so $\langle a \rangle = G$.

equal