recall FTCG.

WARNING: unlike the case for cyclic groups, in general, the converse of Lagrange's thm is false.

even permutations of $\{1,2,3,4\}$

Ex $|A_4| = 12$ but A_4 has no subgroup of order 6.

$\hookrightarrow \frac{4\cdot 3\cdot 2\cdot 1}{2} \leftarrow |S_4|$

Why? Note: A_4 has 8 elements of order 3.

$\hookrightarrow (abc)(d) \rightsquigarrow \frac{4\cdot 3\cdot 2}{3} = 8$

proof by contradiction

Sps $H \leq A_4$ and $|H| = 6$. Then $[A_4 : H] = 2$

so H, aH, a^2H must be redundant for any $a \in A_4$.

so $a \in H$.

Sps $|a| = 3$. Then it must be the case that $aH = H$.

$aH = bH \Leftrightarrow a^{-1}b \in H$

$\hookrightarrow$ if $aH = a^2H$, then $a^{-1}a^2 \in H$,
ie $a \in H$, so $aH = H$.

$\hookrightarrow$ if $H = a^2H$, then $a^2 \in H$ so
$a = a^4 \in H$ so $aH = H$.

$\hookrightarrow$ if $aH = H$, we're done b/c this is what we want.

Conclusion: all 8 elts of order 3 are in H, which has order 6. $\divideontimes$