# External Direct Products

A way to build new groups...

**Defn** Sps $G, H$ are groups. Form a new group, the <u>external direct product of $G$ and $H$</u>,

"G plus H" → $$G \oplus H = \{(g, h) \mid g \in G,\ h \in H\}$$

with binary operation:

$$(g_1, h_1)(g_2, h_2) = (\underbrace{g_1 g_2}_{\text{group operation in } G},\ \underbrace{h_1 h_2}_{\text{group operation in } H.})$$

$G \oplus H = \{(g,h) \mid g \in G, h \in H\}$

groups.

Check: closed? Sps $(g_1, h_1), (g_2, h_2) \in G \oplus H$.

By defn, $(g_1, h_1)(g_2, h_2) = (g_1 g_2, h_1 h_2)$

But $G$ a group, so $g_1 g_2 \in G$.

And $H$ a group, so $h_1 h_2 \in H$.

Thus $(g_1 g_2, h_1 h_2) \in G \oplus H$.

assoc? $(g_1, h_1)[(g_2, h_2)(g_3, h_3)] \stackrel{?}{=} [(g_1, h_1)(g_2, h_2)](g_3, h_3)$

↳ exercise.

identity? Sps $(g,h) \in G \oplus H$. Consider: $(e_G, e_H)$.

Then $(e_G, e_H)(g,h) = (e_G g, e_H h) = (g,h)$ ✓

(similarly $(g,h)(e_G, e_H) = (g,h)$)

inverses? Sps $(g,h) \in G \oplus H$. Consider: $(g^{-1}, h^{-1})$.

exists b/c $G, H$ are groups.

Then $(g,h)(g^{-1}, h^{-1}) = (gg^{-1}, hh^{-1})$

$= (e_G, e_H)$. ✓