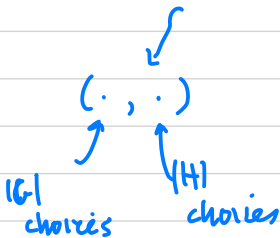


Notes: 1. The external direct product is sometimes denoted  $G \times H$ .

↳ we'll make a distinction for now and use  $G \oplus H$ ... will compare later with notion of internal direct product.

$$2. \quad |G \oplus H| = |G| \times |H|$$



3. We can use the same process to take the direct product of any finite collection of groups:

$$G_1 \oplus G_2 \oplus \dots \oplus G_n = \{ (g_1, g_2, \dots, g_n) \mid g_i \in G_i \}$$

and binary operation is componentwise.

Ex.  $\mathbb{Z}_3 \oplus \mathbb{Z}_6$

$$(1, 4) + (1, 2) = (1+1, 4+2) = (2, 0).$$

$$|\mathbb{Z}_3 \oplus \mathbb{Z}_6| = 3 \times 6 = 18.$$

cyclic? No. Consider any  $(a, b) \in \mathbb{Z}_3 \oplus \mathbb{Z}_6$ . ↙ identity.

$$\hookrightarrow \text{Then } 6 \cdot (a, b) = (6 \cdot a, 6 \cdot b) = (0, 0)$$

So  $|k(a, b)| \leq 6$  for all  $(a, b)$ . ↗ additive notation ...  
like  $[a, b]^6 \dots$   
↘ no one can generate.

Ex.  $\mathbb{Z}_2 \oplus \mathbb{Z}_3$

$$|\mathbb{Z}_2 \oplus \mathbb{Z}_3| = 2 \times 3 = 6.$$

cyclic? yes: consider  $(1, 1)$

↙  $1 \in \mathbb{Z}_2 \dots$  generates  $\mathbb{Z}_2$

↘  $1 \in \mathbb{Z}_3, \dots$  generates  $\mathbb{Z}_3$

2  $(0, 2)$   
3  $(1, 0)$   
4  $(0, 1)$   
5  $(1, 2)$   
6  $(0, 0)$

$\rightsquigarrow (1, 1)$  generates  $\mathbb{Z}_2 \oplus \mathbb{Z}_3$ .