Notes: 1. The external direct product is sometimes denoted $G \times H$.

↳ we'll make a distinction for now and use $G \oplus H$... will compare later with notion of internal direct product.

2. $|G \oplus H| = |G| \times |H|$

$(\cdot\,,\,\cdot)$

$|G|$ choices $|H|$ choices

3. We can use the same process to take the direct product of any finite collection of groups:

$$G_1 \oplus G_2 \oplus \cdots \oplus G_n = \{(g_1, g_2, \ldots, g_n) \mid g_i \in G_i\}$$

and binary operation is componentwise.

Ex. $\mathbb{Z}_3 \oplus \mathbb{Z}_6$

in $\mathbb{Z}_3$ / in $\mathbb{Z}_6$

$$(1,4) + (1,2) = (1+1, 4+2) = (2,0).$$

$$|\mathbb{Z}_3 \oplus \mathbb{Z}_6| = 3 \times 6 = 18.$$

cyclic? No. Consider any $(a,b) \in \mathbb{Z}_3 \oplus \mathbb{Z}_6$.

↳ Then $6 \cdot (a,b) = (6 \cdot a, 6 \cdot b) = (0,0)$ identity.

So $|(a,b)| \leq 6$ for all (a,b). ↳ additive notation... like $[(a,b)]^6$...

↳ no one can generate.

Ex. $\mathbb{Z}_2 \oplus \mathbb{Z}_3$

$$|\mathbb{Z}_2 \oplus \mathbb{Z}_3| = 2 \times 3 = 6.$$

$1 \in \mathbb{Z}_2$ ... generates $\mathbb{Z}_2$

cyclic? yes: consider 1. $(1, 1)$ ↖ $1 \in \mathbb{Z}_3$, ... generates $\mathbb{Z}_3$

2 $(0,2)$
3 $(1,0)$
4 $(0,1)$
5 $(1,2)$
6 $(0,0)$

⤳ $(1,1)$ generates $\mathbb{Z}_2 \oplus \mathbb{Z}_3$.