Ex. In $\mathbb{Z}_{12}$, let $H = \langle 3 \rangle = \{0, 3, 6, 9\}$

and $K = \langle 4 \rangle = \{0, 4, 8\}$.

$H \trianglelefteq G$
$K \trianglelefteq G$.
$G = HK$
$H \cap K = \{e\}$

✓ $H, K \trianglelefteq \mathbb{Z}_{12}$. Why? B/c $\mathbb{Z}_{12}$ abelian, so <u>all</u> subgps normal.

✓ $H \cap K = \{0\}$

additive notation

Finally: $\mathbb{Z}_{12} = H + K$. Why?

some $s, t \in \mathbb{Z}$.

So $\mathbb{Z}_{12} \cong \langle 3 \rangle + \langle 4 \rangle$.

$\hookrightarrow$ $\gcd(3,4) = 1$ so $1 = 3s + 4t \in \underset{H}{\langle 3 \rangle} + \underset{K}{\langle 4 \rangle}$

But then for any $l \in \mathbb{Z}_{12}$, $l = l \cdot 1$

$$= l(3s + 4t)$$

$$= 3ls + 4lt$$

$$\in \langle 3 \rangle + \langle 4 \rangle.$$

✓ So $\mathbb{Z}_{12} \subset \langle 3 \rangle + \langle 4 \rangle$ and thus they are equal as sets.

Note: $\langle 4 \rangle \cong \mathbb{Z}_3$ and $\langle 3 \rangle \cong \mathbb{Z}_4$ so $\mathbb{Z}_{12} \cong \mathbb{Z}_3 \oplus \mathbb{Z}_4$.