$H \trianglelefteq G$
$K \trianglelefteq G$
$H \cap K = \{e\}$
$G = HK$

Ex Let

$$S = \left\{ A = \begin{bmatrix} a & & \\ & a & \\ & & a \end{bmatrix} \;\middle|\; a \in \mathbb{R}, a \neq 0 \right\} \leq GL(3, \mathbb{R}).$$

scalar

center

Since $S = Z(GL(3,\mathbb{R}))$, $S \trianglelefteq GL(3,\mathbb{R})$

Recall: $SL(3,\mathbb{R}) \trianglelefteq GL(3,\mathbb{R})$.

Note: since $\det A = a^3 = 1 \Leftrightarrow a = 1$,

$$S \cap SL(3,\mathbb{R}) = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \text{ (3×3 identity matrix)}$$

Finally, let $X \in GL(3,\mathbb{R})$ with $\det X = c$. $\neq 0$. Then

$$X = \begin{bmatrix} \sqrt[3]{c} & & \\ & \sqrt[3]{c} & \\ & & \sqrt[3]{c} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt[3]{c}} X \end{bmatrix} \in S\, SL(3,\mathbb{R})$$

$\det\left(\frac{1}{\sqrt[3]{c}} X\right) = \left(\frac{1}{\sqrt[3]{c}}\right)^3 \det X = \frac{1}{c} c = 1$

Thus $GL(3,\mathbb{R}) \simeq S \oplus SL(3,\mathbb{R})$.

compare with $GL(2,\mathbb{R})/SL(2,\mathbb{R})$ example.