Factor Groups

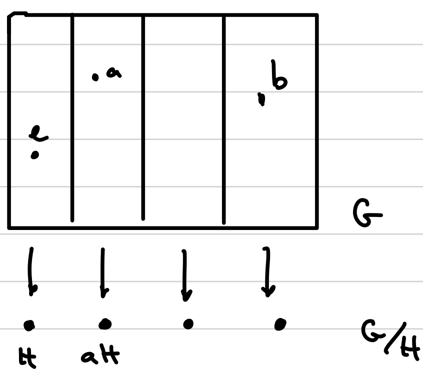
↳ another way to build new groups. Also called quotient groups.

will need a proof

normal!

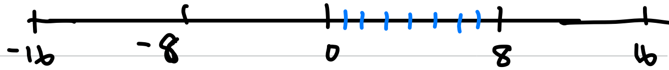
Thm Sps. $H \trianglelefteq G$. The set of distinct left cosets, denoted G/H, is a group under the operation

"G mod H"



$$(aH)(bH) = abH.$$

we'll need to check that this is a well-defined binary operation.



$\underline{\text{Ex.}}$ $\mathbb{Z}/8\mathbb{Z}$ ($\mathbb{Z}$: G, $8\mathbb{Z}$: H)

$\mathbb{Z}$ is abelian so any subgroup is normal.

$H = 8\mathbb{Z} = \{\ldots, -16, -8, 0, 8, 16, \ldots\} = \langle 8 \rangle$

cosets: $H, 1+H, 2+H, \ldots, 7+H$ ← elements of $\mathbb{Z}/8\mathbb{Z}$ a.k.a. $\mathbb{Z}/\langle 8 \rangle$.

H = 8+H

additive notation... like aH.

$\underbrace{(4+H)}_{aH} + \underbrace{(7+H)}_{bH} = \underbrace{(4+7)+H}_{abH} = 11 + H$ — as sets same as $3+H$.

idea: make all multiples of 8 "the same"...

identify them.