$G/H = \{\text{distinct left cosets of } H \text{ in } G\}$.

$(aH)(bH) = abH$

Notes:

index ... # cosets

1. If $|G| = n$ and $|H| = d$, then $|G/H| = [G:H] = \frac{n}{d}$

2. $aH \in G/H$. $|aH|$ can either mean order of aH as an element of G/H or set of set aH. Use context to tell which.

3. Since there is more than one way to represent a given coset, to prove this theorem, we must show the operation is well-defined:

↳ for given input, need unique output.

function $G/H \times G/H \to G/H$.

ie if $aH = cH$ and $bH = dH$, we must show

$$(aH)(bH) = (cH)(dH),$$

ie. $abH = cdH$.

too strong.

↳ Note: we don't have to show $ab = cd$. Rather: $abH = cdH$ as sets.