

Idea of factor groups: Factor out  $H$  by getting rid of "static" from  $H$  ... like a cross-section.

By factoring out different subgroups, get different cross-sectional views of  $G$ .. develop a more clear view of overall group structure of  $G$ .

Ex:  $GL(2, \mathbb{R}) / SL(2, \mathbb{R})$



$GL(2, \mathbb{R})$

$$X SL(2, \mathbb{R}) = Y SL(2, \mathbb{R}) \iff Y^{-1}X \in SL(2, \mathbb{R})$$

$$\iff \det(Y^{-1}X) = 1$$

$$\iff \det X = \det Y$$

So: general element of  $GL(2, \mathbb{R}) / SL(2, \mathbb{R}) = \{ X \in GL(2, \mathbb{R}) \mid \det X = c \}$   
 $(c \neq 0)$

Turns out:  $GL(2, \mathbb{R}) / SL(2, \mathbb{R}) \cong \mathbb{R}^*$

↑ we'll make this rigorous later  
 (1st isomorphism thm)