make note of this example/proof.

Ex $\mathbb{Z}/8\mathbb{Z} \approx \mathbb{Z}_8$

left cosets of $8\mathbb{Z}$ in $\mathbb{Z}$.

proof:

Consider $\varphi : \mathbb{Z}/8\mathbb{Z} \longrightarrow \mathbb{Z}_8$

$$\varphi(a+8\mathbb{Z}) = a \bmod 8.$$

e.g. $3+8\mathbb{Z} = 11+8\mathbb{Z}$

* φ well-defined?

need to check this b/c multiple ways to represent a coset, and formula for φ is based on the representative.

Sps $a+8\mathbb{Z} = b+8\mathbb{Z}$ (NTS: $a \bmod 8 = b \bmod 8$, ie. $\varphi(a+8\mathbb{Z}) = \varphi(b+8\mathbb{Z})$)

Then $a-b \in 8\mathbb{Z}$ (property of cosets)

So $8 \mid a-b$ and thus $a \bmod 8 = b \bmod 8$.

by earlier exercise

Therefore $\varphi(a+8\mathbb{Z}) = \varphi(b+8\mathbb{Z})$, so φ is well-defined.

Now, check φ 1-1, onto, and operation-preserving.

needed for isomorphism.

sps $f(x)=f(y)$.

Show $x=y$.

$\varphi: \mathbb{Z}/8\mathbb{Z} \to \mathbb{Z}_8$

$\varphi(a+8\mathbb{Z}) = a \bmod 8$

φ 1-1 ?

Sps $\varphi(a+8\mathbb{Z}) = \varphi(b+8\mathbb{Z})$,

ie. $a \bmod 8 = b \bmod 8$.

Then $8 \mid a-b$ so $a-b \in 8\mathbb{Z}$, thus $a+8\mathbb{Z} = b+8\mathbb{Z}$. ✓

earlier exercise

properties of cosets.

φ onto? yes. $\mathbb{Z}_8 = \{0,1,2,3,4,5,6,7\}$.

Sps $a \in \mathbb{Z}_8$.

Consider a as an integer.

Then $\underset{\in \mathbb{Z}_8}{a} = \varphi(\underset{\in \mathbb{Z}}{a} + 8\mathbb{Z})$, so φ onto. ✓

$$\left\{ \begin{array}{l} \varphi : \mathbb{Z}/8\mathbb{Z} \to \mathbb{Z}_8 \\ \varphi(a+8\mathbb{Z}) = a \bmod 8 \end{array} \right.$$

φ operation-preserving?

$$\varphi((a+8\mathbb{Z}) + (b+8\mathbb{Z})) = \varphi((a+b) + 8\mathbb{Z}) \quad \text{(addn in } \mathbb{Z}/8\mathbb{Z})$$

$$= (a+b) \bmod 8 \quad \text{(defn of } \varphi)$$

$$\overset{\text{sum in } \mathbb{Z}_8}{=} [a \bmod 8 + b \bmod 8] \bmod 8 \quad \text{(properties of mod arithmetic)}$$

$$= \varphi(a+8\mathbb{Z}) \underset{\text{sum in } \mathbb{Z}_8}{+} \varphi(b+8\mathbb{Z})$$