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Thm Sps $H \trianglelefteq G$, $K \trianglelefteq G$, $G = HK$, and $H \cap K = \{e\}$. Then

$$G \cong H \oplus K = \{(h,k) \mid h \in H, k \in K\}.$$

componentwise binary operation

Remarks

1. Under these four conditions, say G is an <u>internal direct product</u> of H and K.

notation represents internal direct product

2. In this case, we can denote $G = H \times K$, but theorem says we can use $\times$ and $\oplus$ interchangably.

3. $G = HK$ will imply that for every $g \in G$ there exists a pair h, k such that $g = hk$. $H \cap K = \{e\}$ will imply that the pair is unique.

4. This generalizes: Sps. $H_1, H_2, \ldots, H_n \trianglelefteq G$,

$G = H_1 H_2 \cdots H_n$, and

somewhat strong condition. Implies $H_i \cap H_j = \{e\}$ for all $i \neq j$

$H_1 H_2 \cdots H_i \cap H_{i+1} = \{e\}$ for $i = 1, \ldots, n-1$.

Then $G \approx H_1 \oplus H_2 \oplus \cdots \oplus H_n$.